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Several samples so treated worked successfully, and the fact that it was found impossible with U. S. Government 600 minute charcoal suggests that it was due to action of some sort between the hydrocarbons present and the gas. The fact that McBain's work² with hydrogen shows a dual action, and that work done later by others³ on other gases points to a single action, suggests that while surface condensation is likely to explain the major action in both cases, that in the case of hydrogen there must be something else at work as well; either the hydrogen is dissolved or possibly combines with hydrocarbons unsaturated at this liquid air temperature. It is difficult to draw a distinct dividing line between the two, but the latter view is the one taken.

The deposit of inactive carbons on the active base, is according to this theory, much more effective in deactivating for nitrogen, since, interfering with surface condensation, it interferes with the whole effect, this not being the case for hydrogen which has a dual nature. The observations bear this out.

The decided drop in the initial end of the curves in figure 1 at the beginning of the work with hydrogen, which does not appear in the nitrogen curves of figure 2, again shows a difference in the adsorption of these two gases, possibly resulting from an increased fineness of the division of the material with which the hydrogen unites. Further work with other gases will be undertaken.

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¹ These PROCEEDINGS, 5, July 1919, pp. 291-295.

² *Phil. Mag. London*, Ser. 6, 18, 1909 (916).

³ Miss Homfray, *Zs. Phys. Chem.*, 74, 1910 (129).

THE STARTING OF A SHIP

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In this paper we give first the results of a mathematical discussion of the motion of a particle under the action of tangential forces depending only on the velocity of the particle.¹ In the following paragraphs we suggest applications of these results to marine engineering and to the study of the law of resistance of liquids.

We consider a particle, P , moving in any path under the action of forces whose components along the tangent to the path depend on the velocity of the particle alone. We denote the time by t , the distance covered in the time t by x , the velocity at the start by v_0 , at any time t by v , and the acceleration along the tangent by a . By hypothesis, $a = F(v)$. Concerning $F(v)$ we assume

$$F(v) = (\lambda - v)\varphi(v), \quad \lambda > v \geq v_0,$$

and that the function $\varphi(v)$ is not less than a positive number m for such values of v and is finite for $v = \lambda$. It may then be proved

$$(\lambda - v_0)e^{-mt} \geq \lambda - v > 0, \quad (1)$$

$$\lambda t - x = L - \int_v^\lambda \frac{dv}{\varphi(v)}, \quad (2)$$

$$L = \int_{v_0}^\lambda \frac{dv}{\varphi(v)}, \quad \frac{1}{m}(\lambda - v_0)e^{-mt} \geq \int_v^\lambda \frac{dv}{\varphi(v)} > 0. \quad (3)$$

These results may be interpreted as follows: the velocity v increases from v_0 and approaches λ indefinitely as t increases without ever reaching that limit. But this "limiting velocity" λ is, for practical purposes, attained and is hereafter called "full speed." To interpret equation (2) we consider a particle P' describing the same path as P , starting from coincidence with P at the time $t = 0$, and moving uniformly with velocity λ ; then evidently $PP' = \lambda t - x$. Equation (2) shows that PP' increases from zero approaching the finite limit L . We call L the "lost distance" or the distance lost in attaining full speed. It is clear that L is practically the distance that P is behind P' when P has reached full speed. If P is, for example, a ship attaining and then proceeding at full speed, L could be found from observation, for this lost distance is the difference of the distance run in any time τ at full speed and the distance run in the time τ from the start, provided that full speed is attained in the time τ from the start. We introduce the time T lost in attaining full speed, defined by $L = \lambda T$. This lost time T would, in the case of a ship or of a towed model, perhaps be more easily measured than L , for T is the difference of time in running any distance δ at full speed and in running the distance δ from the start, provided that full speed is attained in the distance δ from the start. Then if T is measured L is given by $L = \lambda T$. Now if the acceleration $F(v)$ is known L may be calculated from equation (3). If $F(v)$ is expressed by a formula containing an unknown constant and if L is found by observation the constant may be calculated from (3). These remarks lead to the suggestions of applications to the motion of a ship starting from rest or increasing speed under its own power or to the motion of a towed model.

For a towed model or for a ship increasing speed under the action of the thrust of the screw the component of accelerating force in the direction of the tangent to the path has the form, $f - r$, where f is the tension of the towing line in the case of a towed model or the effective thrust of the screw in the case of a ship, and r is the resistance of the water. No general expression for r is known, but it may be assumed that for any particular model or ship r is a function of v alone. We consider two forms of f : first f is constant, $f = k$; second f is the force exerted by a constant power, $f = c/v$.

Suppose $f = k$. Then if W is the weight of P ,

$$\frac{W}{g}a = k - r(v), \quad a = K - R(v) = F(v).$$

Since $F(\lambda) = 0$, $K = R(\lambda)$, and

$$F(v) = R(\lambda) - R(v) = (\lambda - v)\varphi(v),$$

$$L = \int_{v_0}^{\lambda} \frac{dv}{\varphi(v)} = \int_{v_0}^{\lambda} \frac{(\lambda - v)dv}{R(\lambda) - R(v)}.$$

If we assume the resistance of the water proportional to some positive power of the velocity, $R(v) = K_1 v^n$, the conditions imposed on $F(v)$ are satisfied, and

$$L = \frac{1}{K_1} \int_{v_0}^{\lambda} \frac{(\lambda - v)dv}{\lambda^n - v^n}.$$

If in particular $v_0 = 0$ and if $n = 2$,

$$L = \frac{1}{K_1} \log_e 2 = \frac{0.693}{K_1}.$$

We have the curious result for this particular case that L is independent of λ and consequently of the force k . If L is obtained from observation K_1 is found from the preceding equations and the force required to develop speed λ is determined. Comparison of K_1 for different models would serve to determine the relative merits of their designs.

An apparatus might be simply constructed to give the effect of towing models with a constant force. Suppose, to outline such an apparatus in its simplest form, a model to be towed by a perfectly flexible horizontal cord passing over a pulley without weight or friction on its axis and attached to a descending weight. The tension of the cord would, under the assumptions, be the same on both sides of the pulley, but would vary with the velocity. The equations of motion for the model, weight W , and the descending weight W' are

$$\frac{W}{g}a = T - r(v), \quad \frac{W'}{g}a = W' - T,$$

where T is the tension. Adding the equations we have

$$a = \frac{g}{W + W'} [W' - r(v)],$$

from which it appears that the motion is the same as if the towing force were constant, the force of resistance however being modified by a constant factor. The preceding discussion would be applicable. In a more practical but less simple apparatus consisting of several cords and pulleys, the former not perfectly flexible, the latter neither weightless nor perfectly smooth, the results appear still to be applicable, account being taken of the stiffness of the cords and of the weight of the pulleys, but the fric-

tion of the pulleys on their axles must be eliminated or else the previous results cannot be applied.

If it is found possible so to run the engine of a ship as to deliver constant effective thrusting power at the screw we may give a similar discussion of the motion of the ship, where now $f = c/v$. Even if it is not possible to produce constant effective thrusting power in starting a ship from rest it may be found possible to do so in increasing speed and the following discussion would be applicable. We have

$$\frac{W}{g} a = \frac{c}{v} - r(v), \quad a = \frac{C}{v} - R(v) = F(v).$$

Since $F(\lambda) = 0$, $C = \lambda F(\lambda)$, and

$$F(v) = \frac{\lambda R(\lambda) - vR(v)}{v} = (\lambda - v)\varphi(v),$$

$$L = \int_{v_0}^{\lambda} \frac{dv}{\varphi(v)} = \int_{v_0}^{\lambda} \frac{(\lambda - v)vdv}{\lambda R(\lambda) - vR(v)}.$$

If $R(v) = K_1 v^n$ the conditions imposed on $F(v)$ are satisfied as before. If in particular $v_0 = 0$ and $n = 2$ we have $L = 0.247/K_1$, a value independent of λ and C . The assumption $R(v) = K_1 v^2$ gives $C = K_1 \lambda^3$, a formula connecting power and speed which is often used but which is certainly not correct.²

It is not possible to represent the resistance offered by the water to the motion of a model or a ship by an expression of the form $R(v) = K_1 v^n$. It is suggested that the law of resistance may be studied in two ways: more complicated laws of resistance may be assumed and tested by comparing the measured values of L with the values given by equation (3) under the assumed law. Either constant force or constant power may be used. Secondly, it may appear from experiment that the distance L for a model started from rest and brought to full speed λ , by a constant force, for example, can be expressed as a function of λ . Then the function $R(v)$ determining the law of resistance might be studied from the integral equation

$$L(\lambda) = \int_0^{\lambda} \frac{(\lambda - v)dv}{R(\lambda) - R(v)} = \lambda^2 \int_0^1 \frac{(1 - z)dz}{R(\lambda) - R(\lambda z)}, \quad v = \lambda z.$$

¹ The complete discussion is to be published in the *Annals of Mathematics*, probably in the number for June, 1920.

² See, for example, the "Admiralty Coefficient Formula," $H.P. = \Delta^{2/3} V^3 / K$, given by C. W. Dyson, *Practical Marine Engineering*, 7th edition, 1918, p. 614.